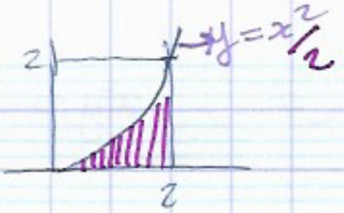
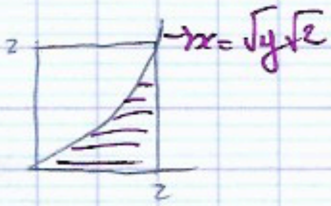
⑤ $I = \iint_D \frac{2x}{(1+x^2+y^2)^2} \, dx \, dy$



$$I = \int_{x=0}^{2} \int_{y=0}^{x^2/2} \frac{2x}{(1+x^2+y^2)^2} \, dy \, dx$$

on pourrait reconnaître un $\frac{u'}{u^2}$ si on avait mis le dx à l'intérieur

$$I = \int_{y=0}^{2} \int_{x=\sqrt{2y}}^{2} \frac{2x}{(1+x^2+y^2)^2} \, dx \, dy$$



$$= \int_{y=0}^{2} \left[\frac{-1}{1+x^2+y^2} \right]_{x=\sqrt{2}\sqrt{y}}^{2} dy$$

$$= \int_{y=0}^{2} \left\{ \frac{-1}{5+y^2} + \frac{1}{1+2y+y^2} \right\} dy$$

formules à connaître

$$\int \frac{dx}{ax^2+bx+c} = \frac{2}{\sqrt{-\Delta}} \arctan\left(\frac{2ax+b}{\sqrt{-\Delta}}\right)$$

(pour $\Delta < 0$)

$$\int \frac{dx}{a+x^2} = \frac{1}{\sqrt{a}} \arctan\left(\frac{x}{\sqrt{a}}\right)$$

$$= \boxed{-\frac{1}{\sqrt{5}} \arctan \frac{2}{\sqrt{5}} + \frac{-1}{3} + 1} \simeq 0{,}340$$